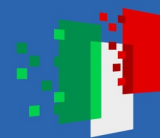




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# Accelerating Heterogeneous Federated Learning with Closed-form Classifiers

Raffaello Camoriano  
*Politecnico di Torino*

*Joint work with:*

Eros Fanì  
Barbara Caputo  
Marco Ciccone

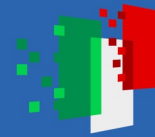
FAIR General Conference, Naples, Italy

September 24<sup>th</sup>, 2024

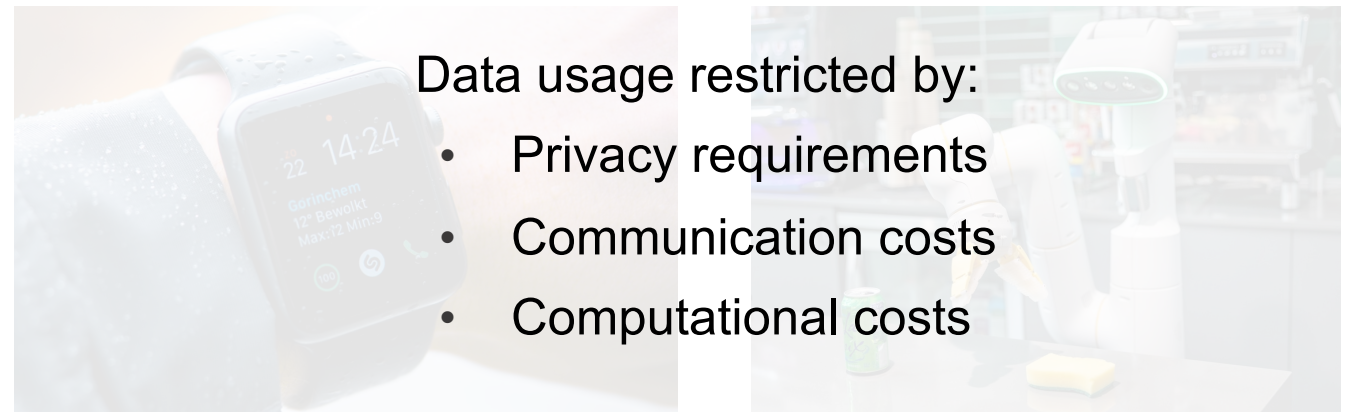
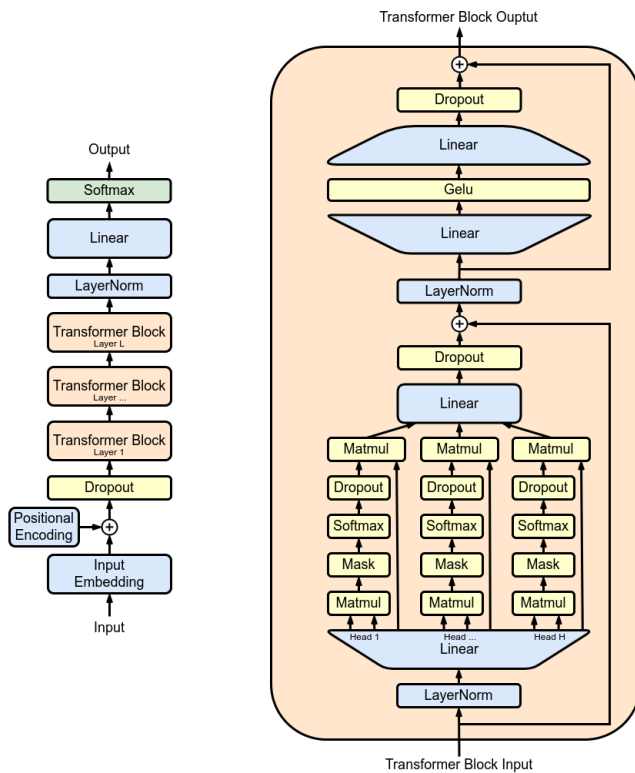


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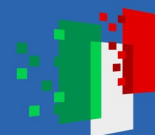


## Leveraging large-scale data on edge devices could improve learning performance



Data usage restricted by:

- Privacy requirements
- Communication costs
- Computational costs

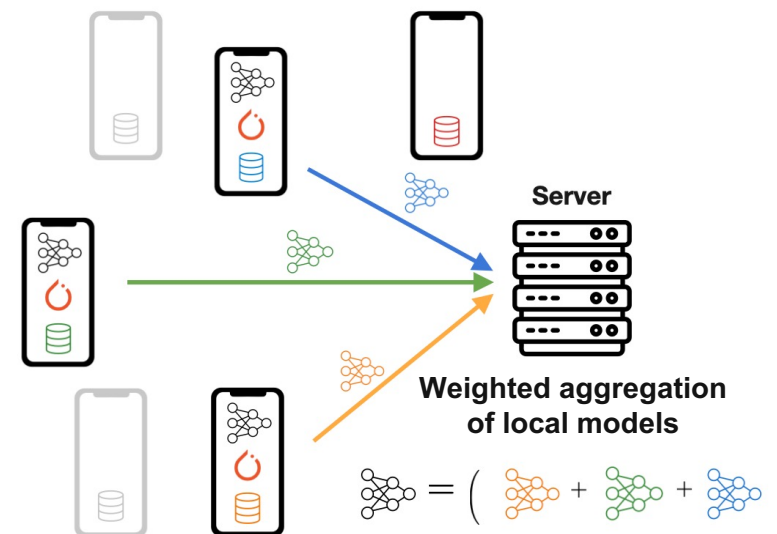


# Federated Learning

• Population of users join a “federation” to train a model collaboratively

- 🔒 Data never leaves the device (privacy preserving)
- 🏠 Training on data is local (distributed computation)
- 📡 Communicate model updates
- 💪 Powerful, but many challenges

• Allows training large-scale models while distributing computations



Federated Training Round

Server



Clients



...



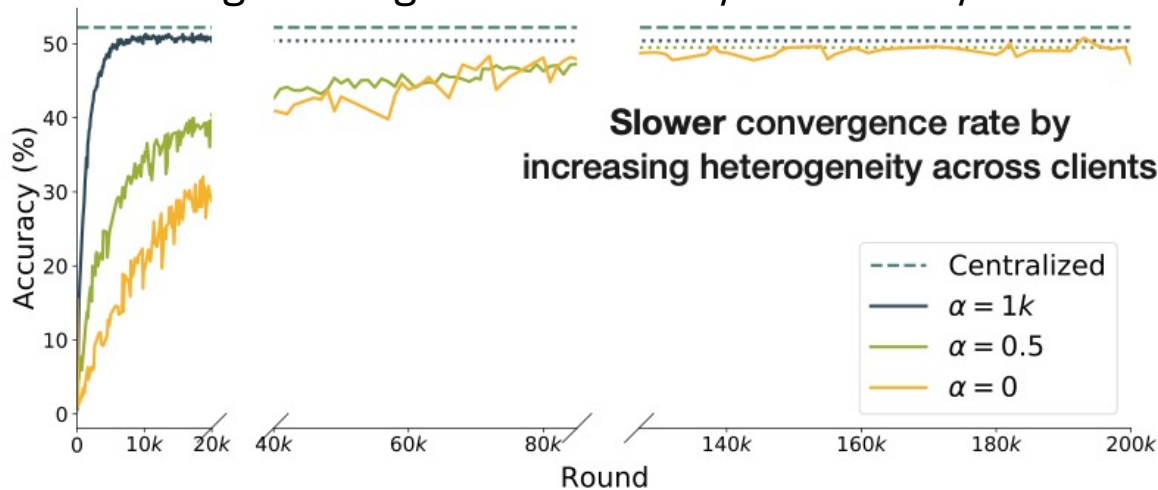
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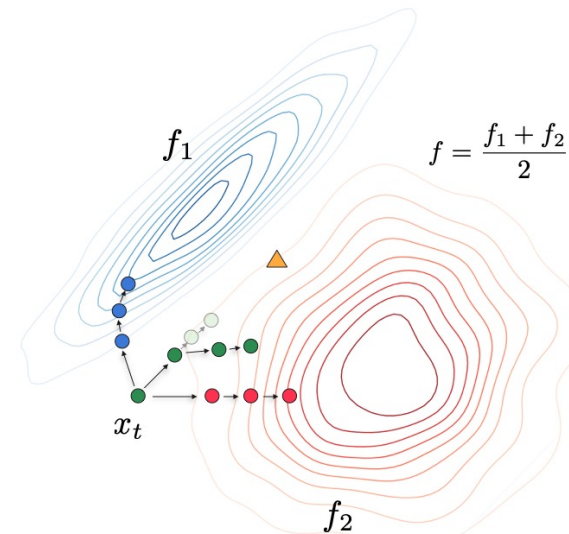
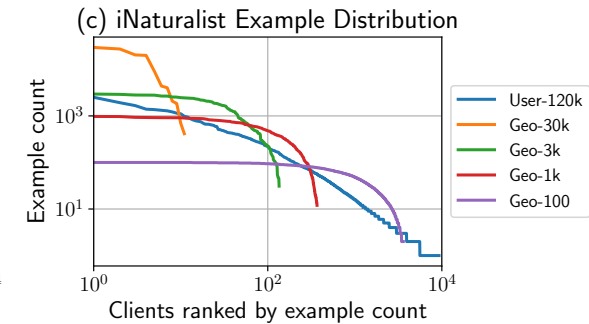
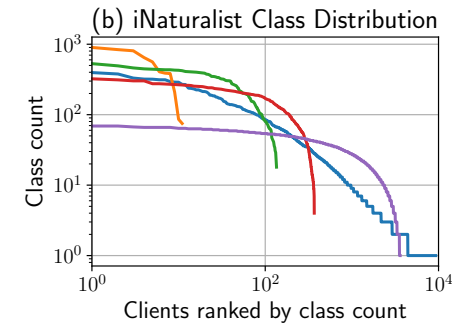


## Challenge of Statistical Heterogeneity

- Samples collected on the edge correlate to specific **user habits, preferences, location** (**Domain Shift, Label Skewness, Size Imbalance**)
- **Clients drift** from global solution by specializing on local data
- Training convergence is severely affected by client drift



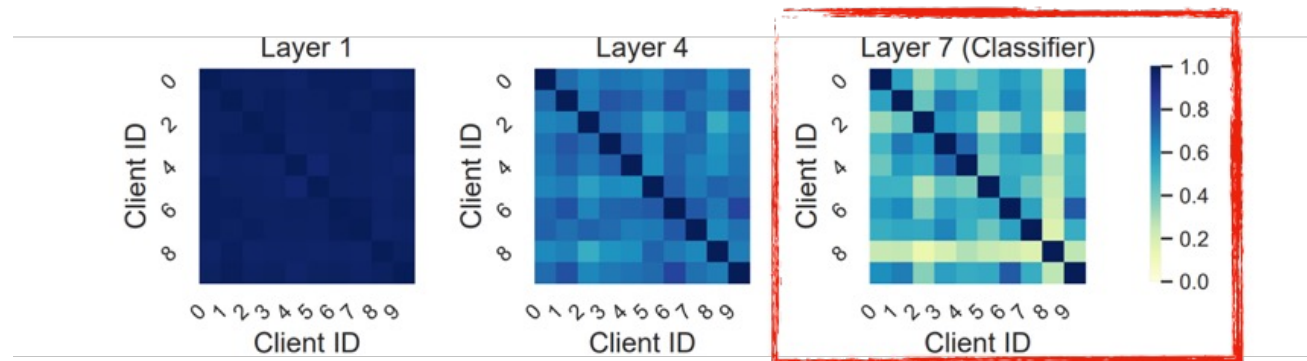
Caldarola, Caputo, Ciccone (FedSAM, ECCV 2022)



Client Drift Problem

## Which part of the model is most affected by heterogeneity?

- Recent work shows that **deeper layers are more subject to client drift** [1]
- Shown by CKA similarities of different layers in clients' local model pairs



- Heterogeneity and partial participation **mainly affect the prediction head** (classifier)

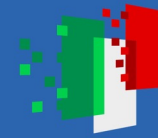
[1] Mi Luo, et al. "No Fear of Heterogeneity: Classifier Calibration for Federated Learning with Non-IID Data". 2021

## Heterogeneity and issues with softmax classifier

- **The Softmax classifier** is sensitive to the direction of gradient updates
  - When **new classes** are added, **probabilities of previously learned classes may diminish**
  - This occurs even if logits of previous classes remain unchanged (because of softmax normalisation)
  - **Classifier is biased**
- Similar effect in Continual Learning

Can we design an efficient FL method  
robust to client drift in heterogeneous settings  
and unaffected by classifier bias?



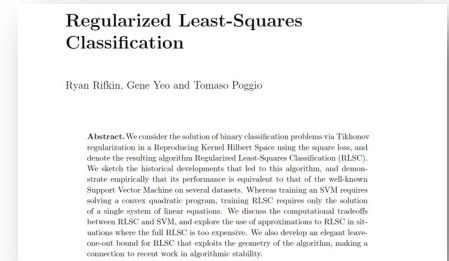


## Background: Ridge Regression for Classification

- We can use a 1-vs-all **Ridge Regression classifier** [2]:  $f(x) = \arg \max_y w_y^\top \psi(x)$ ,

$$W^* = [w_1 \dots w_K] = \arg \min_W \frac{1}{|D|} \sum_{(x,y) \in D} \|W^\top \psi(x) - \text{OneHot}(y)\|^2 + \lambda \|W\|^2$$

- Ridge Regression admits an **exact incremental formulation** [3,4]



Closed-form solution

$$W^* = (\underbrace{X^\top X}_A + \lambda I)^{-1} \underbrace{X^\top Y}_b$$

$$X = [\psi(x_1) \dots \psi(x_N)]^\top$$

$$Y = [\text{OneHot}(y_1) \dots \text{OneHot}(y_N)]^\top$$



**Exact  
Model Updates**

Online Update

$$w_z = (A^{\text{new}} + \lambda I)^{-1} b_z^{\text{new}} \quad \forall z \in \mathcal{Y}$$

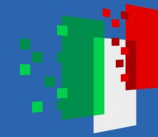
$$A^{\text{new}} = A^{\text{old}} + \psi(x)\psi(x)^\top$$

$$b_z^{\text{new}} = \begin{cases} b_z^{\text{old}} + \psi(x) & \text{if } z = y \\ b_z^{\text{old}} & \text{otherwise.} \end{cases}$$

[2] Rifkin, R., Gene Yeo, and Tommaso Poggio. "Regularized least-squares classification." NATO Science Series Sub Series III Computer and Systems Sciences. 2003.

[3] Camoriano, R.\*, Pasquale, G.\*, et al. "Incremental robot learning of new objects with fixed update time." *IEEE ICRA*. 2017.

[4] Wang, R., Ciccone, M.\*, et al. "Schedule-robust online continual learning." *arXiv preprint arXiv:2210.05561*. 2022.



## Fed3R: Federated Recursive Ridge Regression

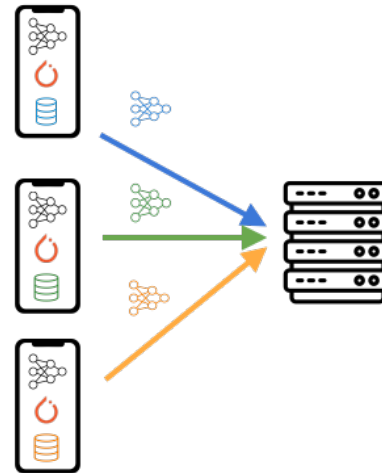
Client side

Compute Local Ridge Statistics

$$\sum_{(x,y) \in \mathcal{D}_k} \varphi(x)\varphi(x)^\top = A_k$$

$$\sum_{(x,y) \in \mathcal{D}_k} \varphi(x)e_y^\top = b_k$$

$e_y \in \mathbb{R}^C$



Server side

Aggregate Statistics

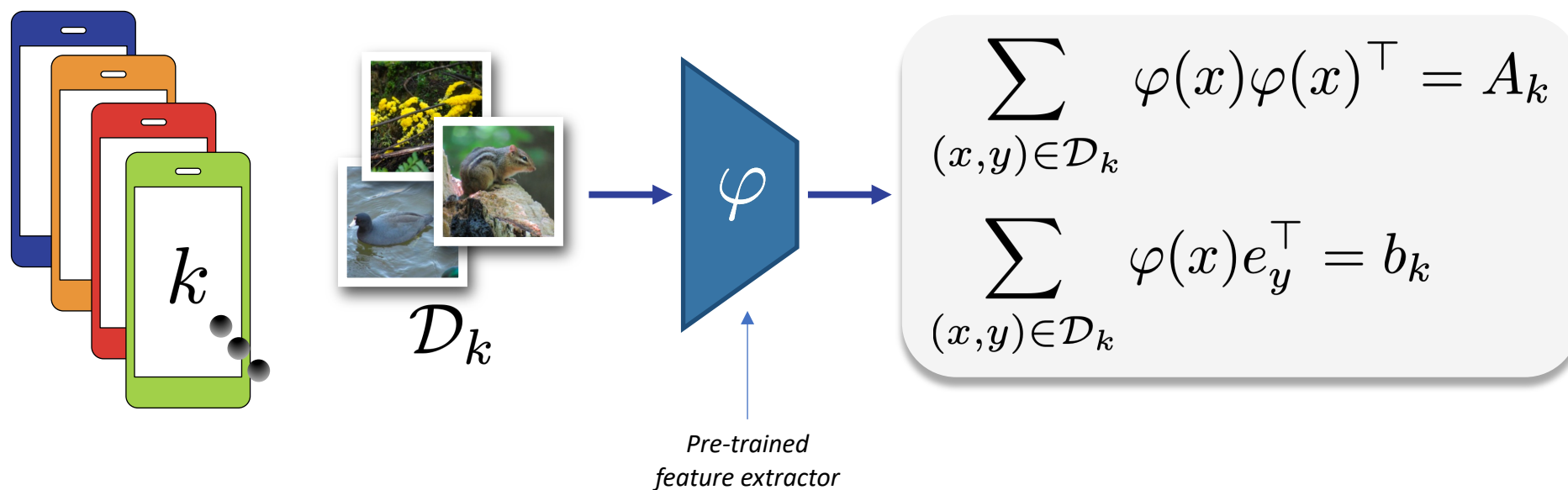
$$A = \sum_{k \in \mathcal{K}} A_k \quad b = \sum_{k \in \mathcal{K}} b_k$$

$$W^* = (A + \lambda I_d)^{-1} b$$

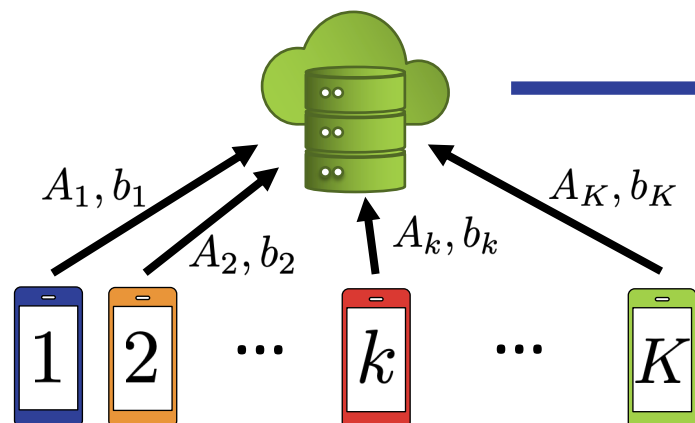
- RR incremental formulation → **Allows for exact aggregation in Fed3R**
  - Immune to statistical heterogeneity
  - Faster convergence
  - Reduced computations and communication
- Fed3R-RF: non-linear variant based on random features for improved accuracy



## Fed3R: Step 1 (client side) - Local computations



## Fed3R: Step 2 (server side) - Exact aggregation



Compute the aggregate statistics

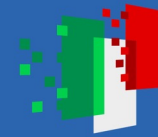
$$A = \sum_{(x,y) \in \mathcal{D}} \varphi(x)\varphi(x)^\top = \sum_{k \in \mathcal{K}} \sum_{(x,y) \in \mathcal{D}_k} \varphi(x)\varphi(x)^\top = \sum_{k \in \mathcal{K}} A_k$$

$$b = \sum_{(x,y) \in \mathcal{D}} \varphi(x)e_y^\top = \sum_{k \in \mathcal{K}} \sum_{(x,y) \in \mathcal{D}_k} \varphi(x)e_y^\top = \sum_{k \in \mathcal{K}} b_k$$

Centralized dataset

Local datasets for each client  $k$   
(Valid for any federated split)

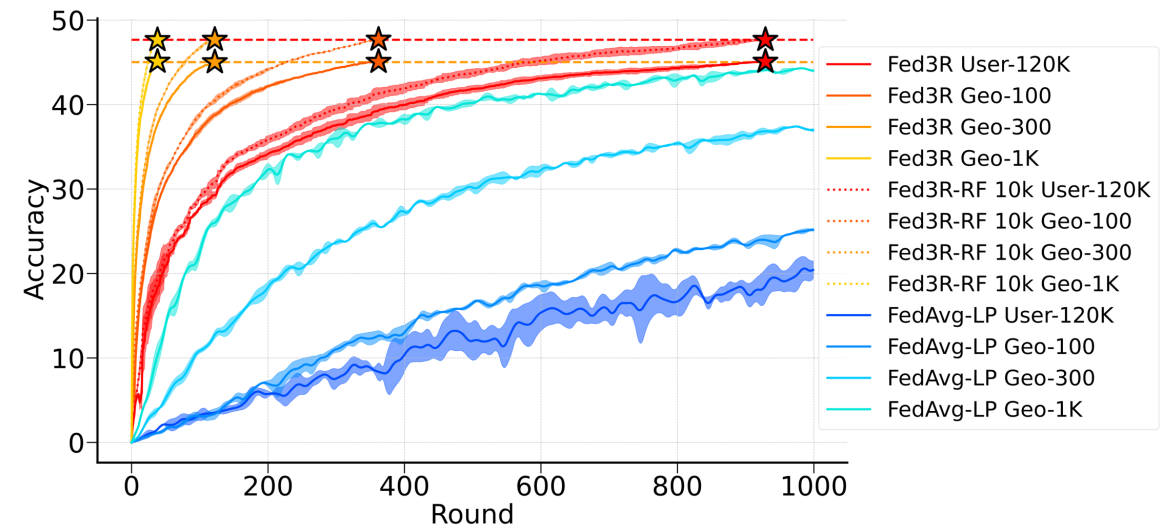
Closed-form RR solution yields exact aggregate classifier:  $W^* = (A + \lambda I_d)^{-1}b$



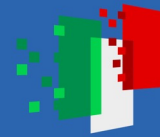
## Fed3R is immune to statistical heterogeneity

- Immune to heterogeneity: equivalent to exact centralized solution
- Convergence guaranteed in a single pass over clients
- Memory and computationally efficient

	User-120K	Geo-100	Geo-300	Geo-1K
# clients	9275	3606	1208	368
# classes	1203	1203	1203	1203

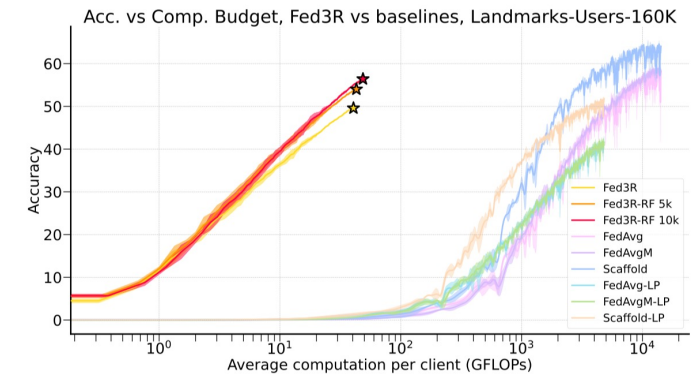
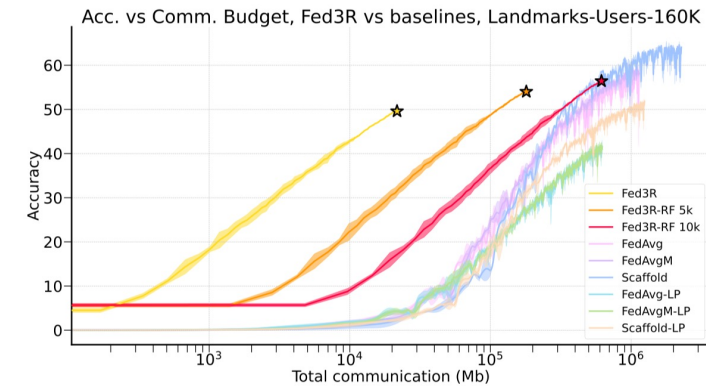
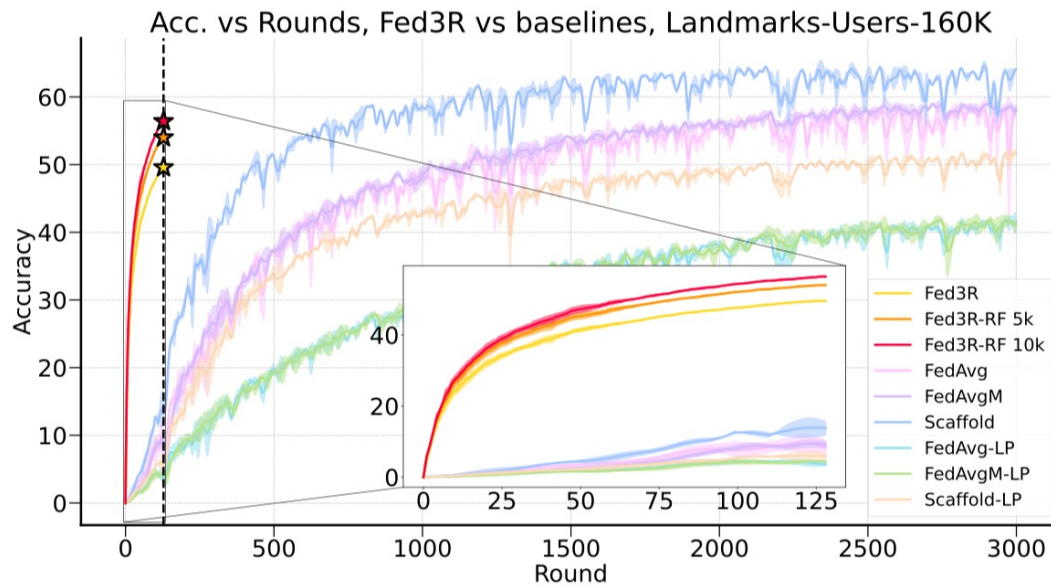


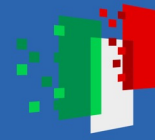
Fed3R performance is invariant to different federated splits of **iNaturalist** (Hsu, 2020).  
(MobileNetV2 pretrained on ImageNet-1k)



# Experiments with fixed feature extractor Fed3R & Fed3R-RF vs. baselines

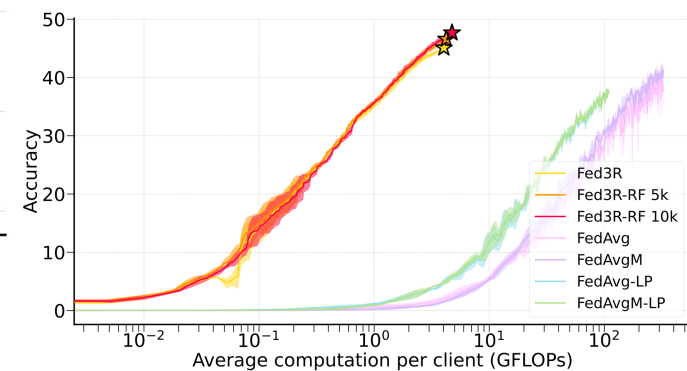
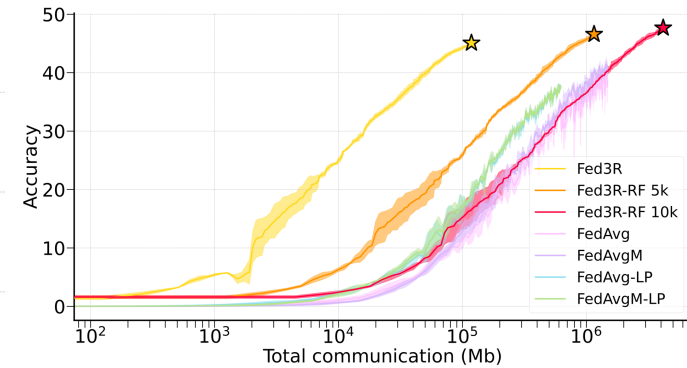
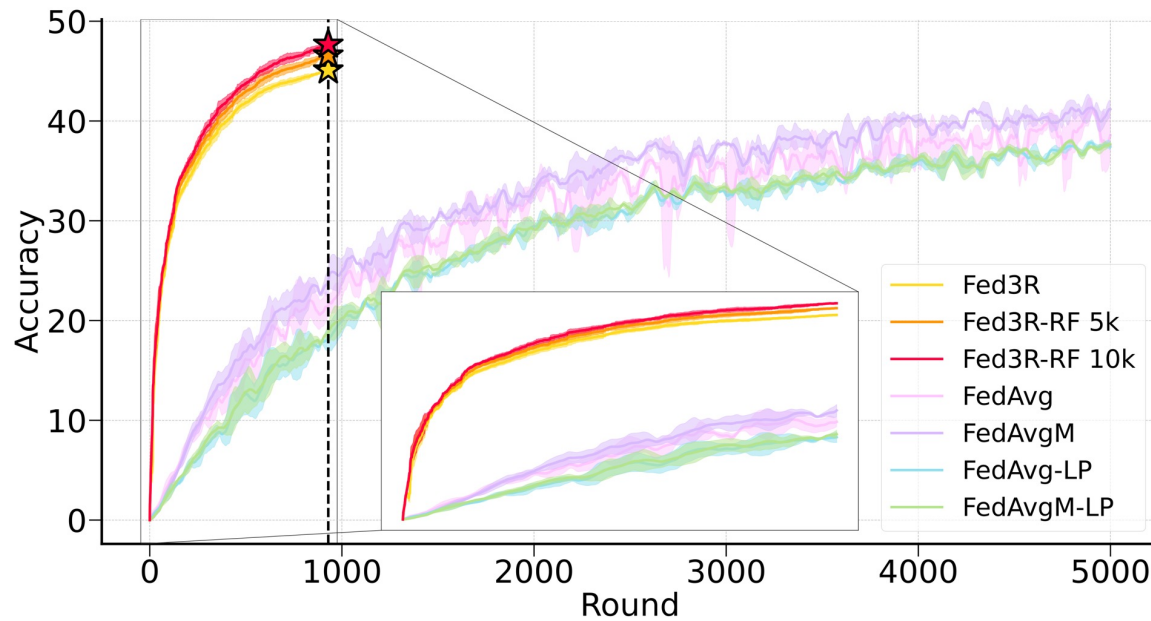
Google Landmarks Dataset v2 - 2028 classes – 1262 clients

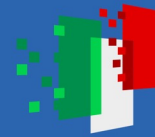




## Experiments with fixed feature extractor Fed3R & Fed3R-RF vs. baselines

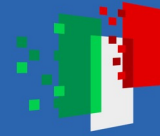
iNaturalist-Users120K - 1203 classes – 9275 clients





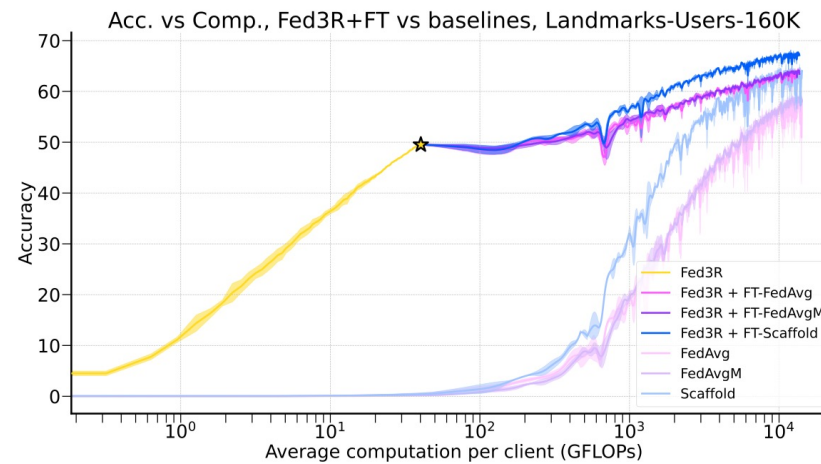
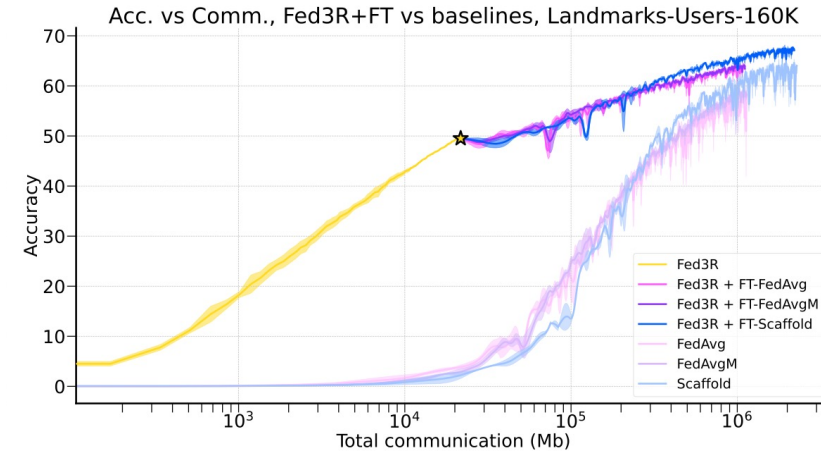
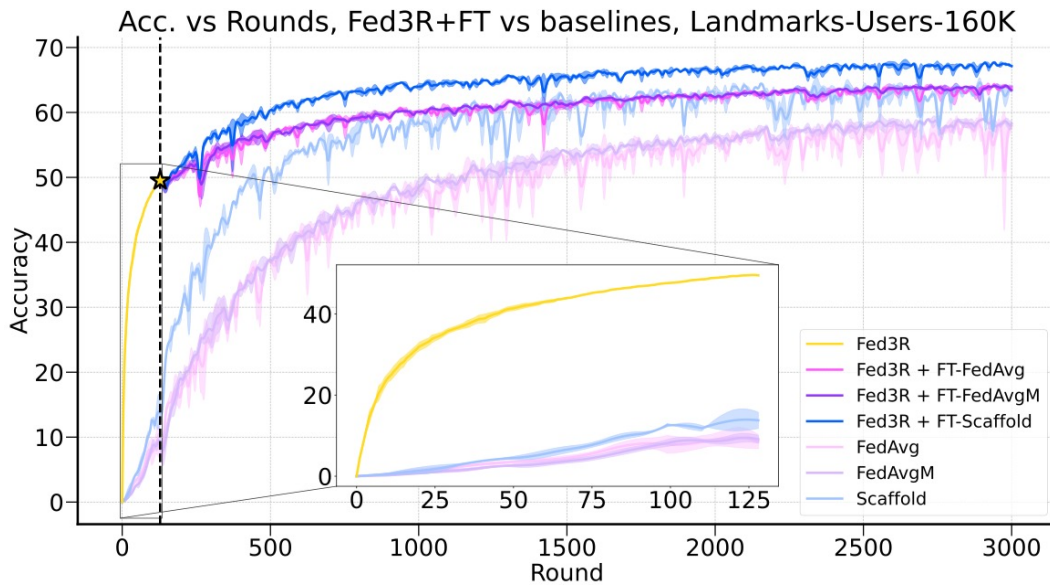
## Fed3R-FT: Fine-Tuning variant

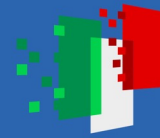
- Fed3R can also be used as robust initialization for *any* FL algorithm
- **Fed3R+FT variant:** last layer initialization for faster fine-tuning
  - Fed3R+FT: fine-tune the whole model
  - Fed3R+FTlp: fine-tune only the classifier
  - Fed3R+FTfeat: fine-tune only the feature extractor
- Learning or fine-tuning in extreme cross-device settings is hard:
  - We show that **fine-tuning further improves final accuracy**
  - Fixing Fed3R and fine-tuning only the representation **stabilizes training**



# Experiments with Fine-Tuning

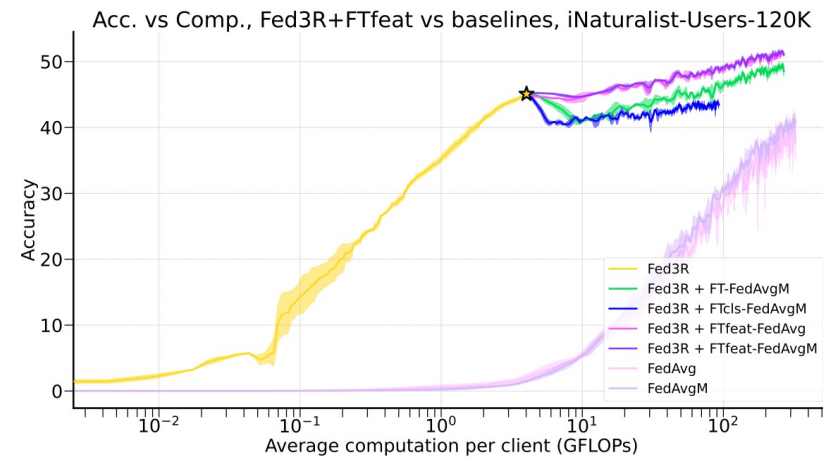
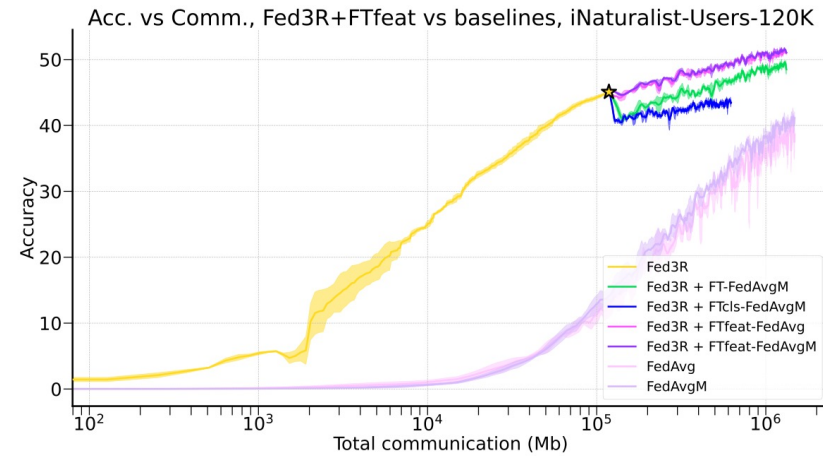
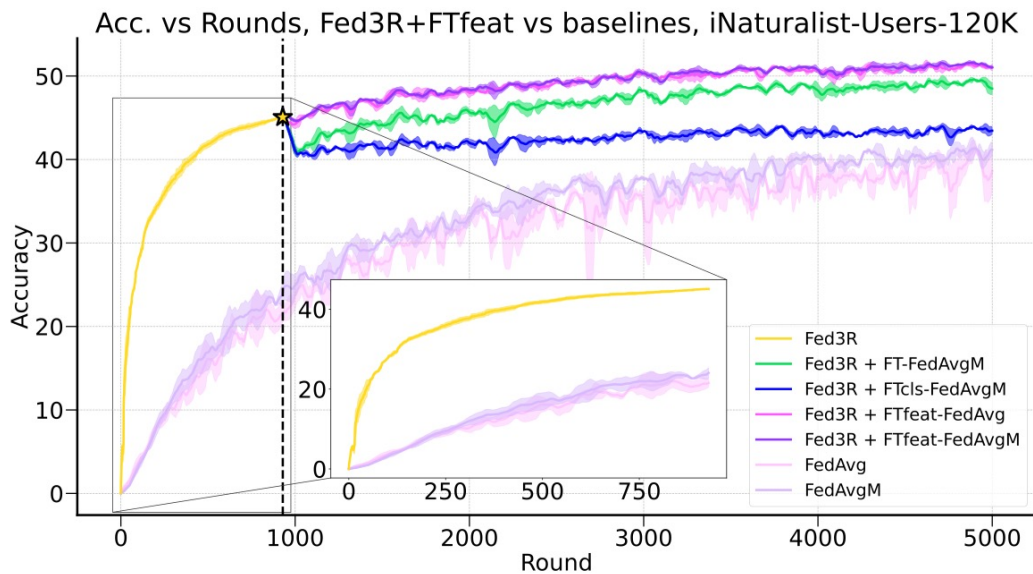
Gldv2 - 2028 classes – 1262 clients



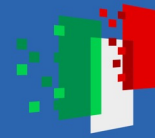


# Experiments with Fine-Tuning

iNaturalist - 1203 classes – 9275 clients







## Conclusions

- We introduce **Fed3R**, a robust FL algorithm with exact aggregation
  - Faster convergence than softmax-based classifiers
  - Communication and computationally efficient
- Fed3R-RF enables accuracy/communication cost trade-off
- Fed3R-FT yields improved accuracy and more stable training in cross-device settings

## Next steps

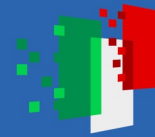
- Extension of Fed3R to the Personalized Federated Learning setting:
  - Fed3R model training
  - Additional fine-tuning on individual devices using client data for improved local performance (data stays private)



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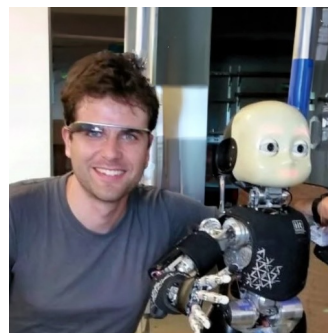
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*ICML 2024, Wien, Austria*

*<https://fed-3r.github.io/>*



*Contact: [eros.fani@polito.it](mailto:eros.fani@polito.it)*